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Golden Proportion in Computational Analysis: Reflecting the Universal Beauty

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Abstract: **Numerous mathematics concepts or figures in human history had significantly improved in reputation as a result of their obscure meanings. Golden proportions and other associated ideas, including the golden ratio, are one of the concepts that had received an extraordinary breakthrough in terms of their recognition received by humans, due to the spiritual characteristics of golden proportions' physical existence. Under the general umbrella of golden proportions encloses concepts such as the golden ratio, Golden geometric shapes, golden angle, and the Fibonacci sequence. Often referred to as the golden section, divine proportion, golden ratio or golden mean in Mathematics, is known as an irrational number, equivalent to 1.618, represented by phi (Φ), a Greek symbol. Golden ratio is found across all of nature, such as animals, plants, weather, solar systems, and even in the universe. The application of the Fibonacci sequence can be witnessed in various fields, which including architecture, design, medical, environment and science. Continuous of this, the golden ratio was applied in various computer science applications, such as the development and analysis of, data structures, and algorithms and solving mathematical problems. It is mainly involved in data compression, sorting algorithm, generating random numbers, and connecting points on a computer display. This review is discussed the golden ratio in nature and its applications in the area of computer science and mathematics.**

Keywords: **Natural beauty, Fibonacci sequence, golden ratio, Mathematics.**

I. INTRODUCTION

There is a unique ratio, which can be used to explain the proportions of everything from nature's tiniest atoms to the advanced patterns in the universe. To maintain the balance, nature relies on these proportions, scientist called this 'golden proportion' [1], [2]. Under the general umbrella of golden proportions encloses concepts such as the golden ratio, golden geometric shapes, golden angles, and the Fibonacci sequence. The golden ratio is 1.618, denoted by 'phi' (Φ), the Greek letter [3]. The golden ratio originated from the Fibonacci numbers, a series of numbers where each entry is the sum of the two preceding numbers. In other words, when a line segment is divided into two portions of different lengths, the ratio of the entire segment to the longer part is the same as the ratio of the longer part to the shorter half, this is known as the "golden ratio." [4], [5]. The golden ratio is more accurate and can be portrayed on a line separated into 2 segments of various lengths [longer length (a) and shorter length (b)] in a fashion that $a/b = (a+b)/a$ = Golden Ratio (1.618) [6], [7] (Figure 1a). As the golden ratio is an irrational value, the digits following the decimal are never-ending. Originally, the golden ratio has been mentioned as the "extreme and mean" ratio in all elements as recognized by Euclid. Two hundred years later, this "dividing" or "sectioning" was properly identified by the Ancient Greeks. Following this, both this "ratio" and "section" became designated as Golden in 1835 by Martin Ohm, a German mathematician.

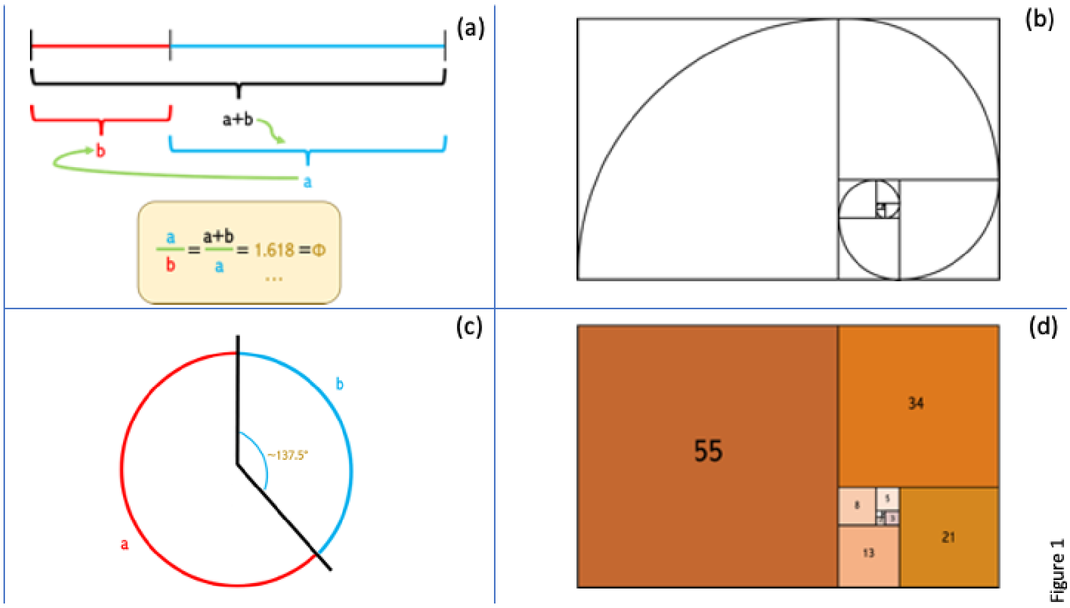

Figure 1: (a) The Golden Ratio's more accurate value can be portrayed on a line separated into 2 segments of various lengths [longer length (a) and shorter length (b)] in a fashion that a/b = (a+b)/a = Golden Ratio (1.618). (b) Subsequent formations of squares within ensuing Golden Rectangles construct the result of the initial Golden Rectangle being completely occupied by squares, its size diminishing following each step. When each square's diagonals join together through an arc, the geometric shape of a "Golden Spiral" is formed. (c) "Golden Angle", 137.5° is a concept where its emergence was according to the separation of the circumference of a circle into larger (a) and smaller (b) arcs in a manner such that the proportion follows a/b = (a+b)/a = Golden Ratio (1.618). The angles the diagram represents comply with this equation, in which the angle of the larger arc is 222.5°, while the smaller arc has an angle of 137.5°. (d) Fn+1 = Fn + Fn−1, the Fibonacci Sequence follows a series of numbers in which each number is the addition of the 2 preceding numbers. Therefore, the sequence goes as such: 0, 1, 1, 2, 3, 5, 8, 13, 34, 55....

II. FIBONACCI SEQUENCE

Besides this, particularly geometric shapes generally established based on the golden proportions' rules are globally known as "Golden Geometric Shapes". Contextually, a "Golden Rectangle" refers to a particular geometric shape in which its length to width is equivalent to the Golden Ratio. It is also widely acknowledged to have highly visually attractive aesthetics [8]. This rectangle can be portioned into a square and an additional golden rectangle that could afterward be split into another square and golden rectangle too. In simpler terms, subsequent formations of squares within ensuing golden rectangles construct the result of the initial golden rectangle being completely occupied by squares, its size diminishing following each step [9], [10]. When each square's diagonals join together through an arc (1/4th of a circle), the geometric shape of a "Golden Spiral" is formed (Figure 1b) [11]. Approximate or true Golden Spirals are widely known for their manifestation in certain galaxies, human ears, and seashells. Various other geometric shapes such as pentagrams, or at times, triangles, may also have the golden proportions rule applied to them as well. Moreover, the "Golden Angle", 137.5° is a concept where its emergence was according to the separation of the circumference of a circle into larger (a) and smaller (b) arcs in a manner such that the proportion follows a/b = (a+b)/a = Golden Ratio (1.618). The angles the diagram on the left represents comply with this equation, in which the angle of the larger arc is 222.5°, while the smaller arc has an angle of 137.5°. A variety of botanic structures such as branches of plant stems and leaves represent this "Golden Angle" (Figure 1c). Finally, given by Fn+1 = Fn + Fn−1, the Fibonacci Sequence follows a series of numbers in which each number is the addition of the 2 preceding numbers [12]. Therefore, the sequence goes as such: 0, 1, 1, 2, 3, 5, 8, 13, 34, 55.... The Hindu-Arabic numerical system (presumed Fibonacci Sequence) was used for the formation of the Ancient Sanskrit, which was followed by Leonardo of Pisa (or Fibonacci), an Italian scientist, who came up with a nickname for the Fibonacci numerical system, which ended up becoming the "Fibonacci Sequence." Sequential Fibonacci numbers have ratios approaching phi [13]. (Figure 1d).

III. GOLDEN PROPORTIONS IN NATURE

The universe is unpredictable and chaotic, at the same time it is an organized physical realm surrounded by various mathematics laws, and the golden ratio is one of them [14]. The appearance of the golden ratio is found in many obvious and notable items in nature, such as flowers, fruits, and human body parts [15]. For example, the Fibonacci sequence can be witnessed in effect when relating it to the petal count of flowers the majority of the time (Figure 2a). For example, Within the center of sunflowers, a marvel of the seeds inside can be identified; the 2D pattern formed follows the Fibonacci sequence 2 spirals, also known as parastichies, are constructed, with one spiral being emanating from the center in an anti-clockwise direction, the other in a clockwise direction (Figure 2b). As can be seen in the illustrations above, the number of anticlockwise spirals is 21, while there are 34 clockwise spirals. 21 and 34 are adjacent numbers in the Fibonacci sequence. Similarly, most of the human body parts are arranged as per the Fibonacci sequence [5], [16], such as the height of the human being over the distance between the navel and the foot, The distance between the top of the head to the shoulder line over the face length, The distance between the top of the head to the navel over the distance between the top of the head to the shoulder and Distance of the navel to the knee over the distance between the knee to the end of the foot (Figure 3). Not only outside, researchers found that the inside part of the body, like the rhythm of the heart and the structure of DNA, also follow the Fibonacci numbers [17], [18].

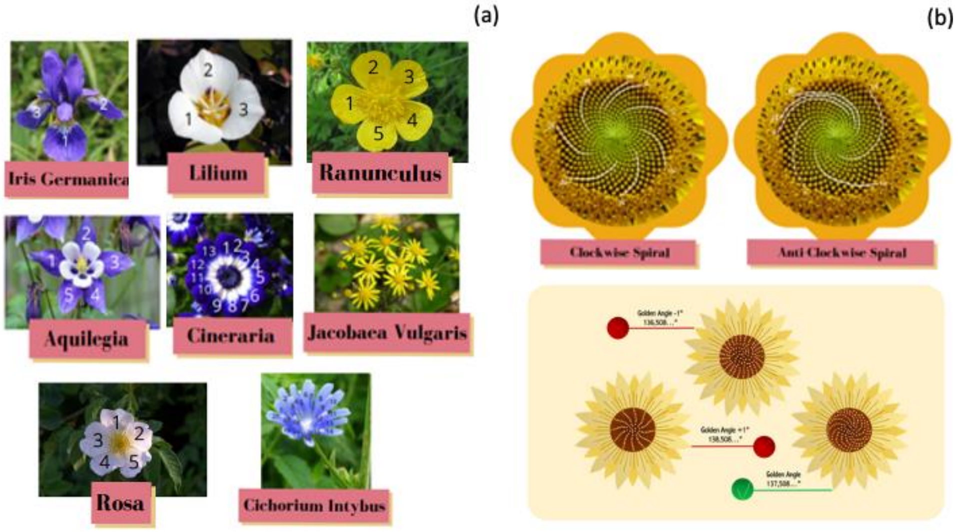

Figure 2: The appearance of golden ratio in nature. (a) The application of the Fibonacci sequence can be witnessed in effect when relating it to the petal count of flowers most of the time. (b) Within the centre of sunflowers, a marvel of the seeds inside can be identified; the 2D pattern formed follows the Fibonacci sequence. (c)

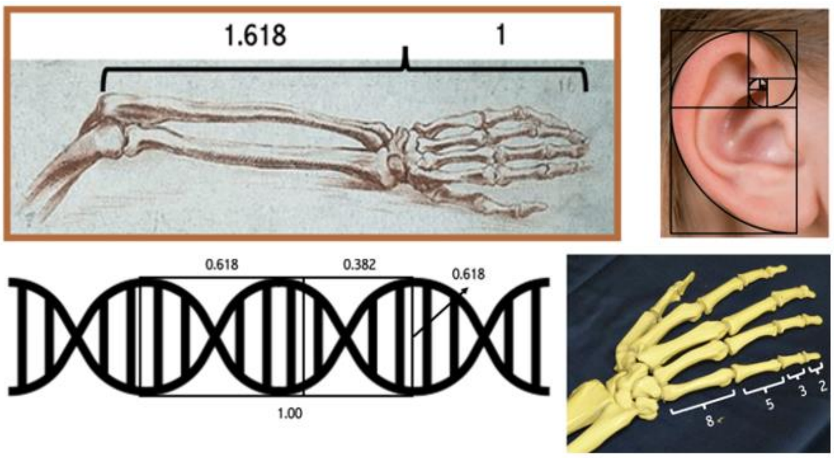

Figure 3: Human body parts are arranging as per Fibonacci sequence.

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IV. APPLICATIONS OF GOLDEN PROPORTIONS IN COMPUTER SCIENCE

Fibonacci numbers have become popular to introduce a recursion for computer science students. Apart from that, Fibonacci exists anywhere within computer science such as in algorithms and data structures. Fibonacci heaps and Fibonacci search. The two main examples are Fibonacci heaps and Fibonacci search. Fibonacci heaps is a data structure, which contains a collection of trees and follows a max or min heap property [19]. The tree is constructed in a way such that Fn=Fn-1+Fn-2, n≥ 2, F0=1; F1=2. Fibonacci heaps maintain the pointer to the minimum value and all the roots are connected with a circular doubly linked list, all of them can be easily accessed with single minimum pointer (Figure 4). Fibonacci heaps are utilized to implement the priority queue in Dijkstra's algorithm, giving the algorithms a very effective running time with a less rigid structure. In general Fibonacci heap helps to reduce the time complexity of the decrease key and has importance in the prim and Dijkstra algorithms. With the normal binary heap, the time complexity of the algorithm is O(VLogV + ELogV) and it is improved to O(VLogV + E) when using the Fibonacci heap (Table 1). Researchers used the Fibonacci heap in the algorithm to reduce the computational cost in the search for the smallest network route [20]–[22]. They use the method to create the mobile transmission network and found that the method reaches a lower transmission power compared with the normal device settings. Similar to Fibonacci heap, Fibonacci search is a method of searching a sorted array using a conquer and divide algorithm, which narrows down the possible locations with the help of Fibonacci numbers [23], [24]. They are used in various aspects of computer science such as used in magnetic tapes, used for large arrays that cannot fit in the RAM or CPUcache. Apart from that, the Fibonacci search gives better efficiency than the usual binary search. Researchers used the real-time model free minimum seeking autotuning method assisted with the Fibonacci search algorithm to develop an unmanned aerial vehicle [25]. This approach gives lesser computational complexity and does not need any dynamics model to obtain the proper tuning of a controller. In another research a beamformer design was developed based on the Fibonacci search. They performed beamforming for a ULA (Uniform linear array) based on an algorithm utilized with Fibonacci search. The performance is compared with other heuristic optimization and the results proved the superiority of the Fibonacci search in locating a solution with higher precision [26]. Apart from that Fibonacci is helping in various aspects of computer science applications. Following is some of the application.

Table 1: Comparison between Binary and Fibonacci Heap

Operation	Binary Heap	Fibonacci Heap
Insert	0(log N)	0(1)
Find minimum	0(1)	0(log N)
Delete	0(log N)	0(log N)
Decrease Key	0(log N)	0(log N)
Union	0(N)	

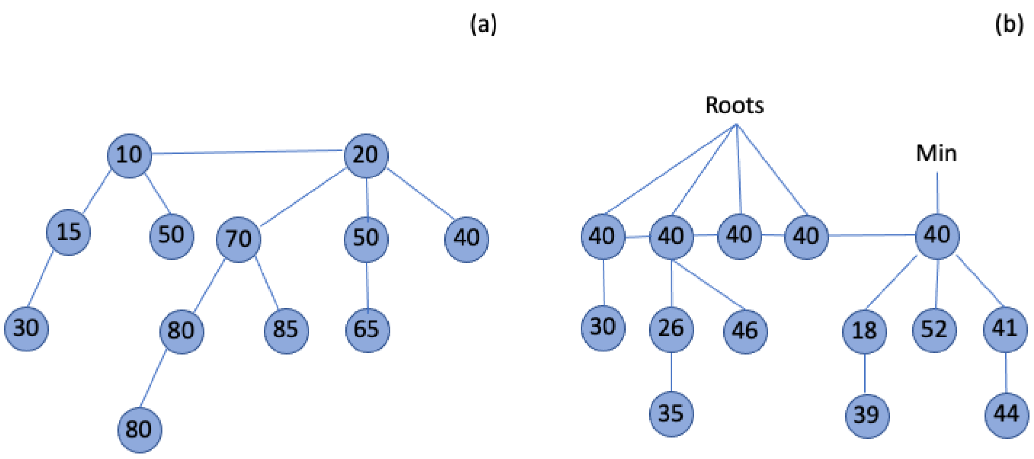

Figure 4: (a) Binomial heap; (b) Fibonacci Heap. Fibonacci heaps maintain the pointer to the minimum value and all the roots are connected with circular doubly linked list, all of them can be easily accesses with single minimum pointer.

A. Fibonacci Sequence in Cryptography

Covering investigations of protocols and algorithms for safe and secure information, cryptography is a significant aspect of information security [27]. As technology advances, cryptographic algorithm designs are commonly improved to guarantee that information is safe. When it comes to safeness, the question is always whether or not these algorithms are secure to the extent that they can safeguard significant information [28]. So far, the most significant and prominent element to deliver high-level security is block ciphers [29]. In general, block ciphers are deterministic algorithms on an established length group consisting of bits often called blocks. An established length block of plaintext message blocks is transformed into same-length cipher text blocks through the algorithms. Due to block ciphers being widely studied and successful, based on the AES substitution box (known as S-box), the block cipher was converted into a modernized design. The purpose of this design was to substitute blocks of input bits, receiving a group of output bits. However, there are issues with the current S-box design. The first issue relates to a good S-box's design, while the second focuses on a given S-box's verification as one cryptanalytical technique. Hence, being able to construct safe S-boxes and make them so that they can be utilized in various cryptosystems for improved security is a current issue [30]. Now, no algebraic procedures that can provide the complete and preferred set of S-box properties are available. Nevertheless, in a study, it was portrayed that secure communication can be made from cryptanalysis attacks through the usage of the Fibonacci sequence [31]. The sequence has the capability of fulfilling requirements for communication such as robustness, security, and capacity to allow secureness when transmitting data over an open channel. It is also shown through recent studies that encryption and decryption algorithm through the usage of the Fibonacci sequence is quicker in comparison to RSA algorithms and symmetric algorithms [32], [33].

Going into further detail, cryptosystems have reliance upon the assumption that since numbers of mathematical problems cannot be solved in polynomial times, they are computationally intractable. The Fibonacci sequence is a numbering system naturally witnessed in many aspects of nature, may it be the number of petals on a flower, the arrangement of leaves in plants, DNA structure, the human face, and more. In computer science, a foundation for multiple algorithms is created through Fibonacci numbers. In a previously conducted study, the application of the Fibonacci sequence in encryption and decryption algorithms was tested. The results showed that the initial message contents were altered to the ciphertext by transforming every character from the message according to Fibonacci numbers. An example may be witnessed below.

i.Encryption and Decryption Method

If the message, "CODE," were to be encrypted, an unsecured channel would send the message. Based on the Fibonacci sequence, the security key is chosen [27], [34]. To generate a ciphertext and allow the Fibonacci sequence to be incorporated, as a first security key, any 1 character is chosen. For example, if the first security key were to be k:

Plain text: CODE

Characters: k l m o p q r s t u v w x y z a b c d e f g h i j k l….

Fibonacci: 1 2 3 5

Cipher Text: k l m o

Converted to Unicode symbols, the ciphertext is saved in a file. Through a transmission medium, the file is transferred. This is the first level of security. As for the second level of security, the ASCII code of every character is obtained from both the ASCII code of its previous character and the ciphertext. Following this, the following character is added to the ASCII code of the synonymous character from the original message. As a security key to continue encoding the characters available to Unicode symbols from the ciphertext, the ASCII codes of four characters are utilized [14].

For instance: ASCII Code of Previous Character + ASCII Code of Synonymous Character + ASCII Code of Following Character + ASCII Code of Cipher Code Character -> SUM

k = 106(j)+107(k)+108(l)+67(C)

l = 107(k)+108(l)+109(m)+79(O)

m = 108(l)+109(m)+110(n)+68(D)

o = 109(m)+111(o)+112(p)+69(E)

Just from observing the symbols in a text file, it would not be possible for an unknown person to identify the meaning, resulting in the message remaining confidential, unless the retrieval procedures are known. At present, there are three major types of encryption algorithms such as symmetric encryption algorithm, one-way hash algorithm, and asymmetric encryption algorithm. One was a hash algorithm, also called the Hash algorithm, which is a function that compresses messages of arbitrary to a fixed length. In a symmetric encryption algorithm, the decryption and encryption keys are identical. Examples of symmetric encryption algorithms are RC4 (Rivest Cipher 4), DES (Data Encryption Standard), and AES (Advanced Encryption Standard). The asymmetric encryption algorithm, also known as Asymmetric Key Cryptography, used two keys to perform, which include a private key and a public key. If the public key is used to encrypt data, only the associated private key may decrypt it. Only the associated public key can be used to decrypt material that has been encrypted using the private key.

The process of decryption is just the opposite of encryption. The person receiving the symbols would extract every symbol from the text file and would soon after map it to figure out its hexadecimal value [35]. The value obtained could be converted into a decimal value in which the key can be used to identify the plain text (Figure 5). Without having prior knowledge of the key, an unknown person cannot decrypt the symbols and understand any secret message that is supposed to be delivered. Through these studies, the Fibonacci sequence may hold the key to improving block cipher performances in cryptosystems.

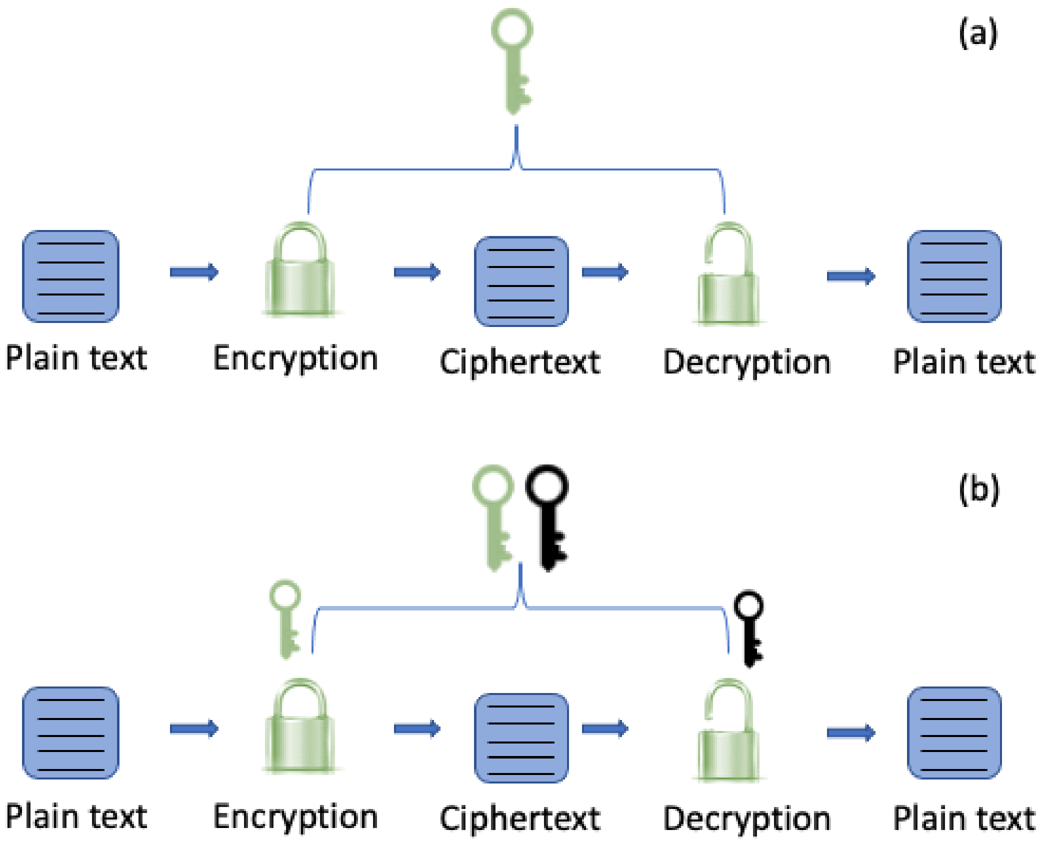

Figure 5: (a) Symmetric encryption algorithm; (b) Asymmetric encryption algorithm. In symmetric encryption algorithm, the decryption and encryption key are identical. Asymmetric encryption algorithm used two keys to perform, which including private key and public key. If the data is encrypted with the public key, only the related private key can be decrypted. If the data is encrypted with the private key, only the related public key can be decrypted.

B. Fibonacci Sequence in Artificial Intelligence

Neural networks, also known as artificial neural network, is the engineering and science involving the usage of computers to understand human intellect. However, most of nature, including the structures within the human, obey the Golden Ratio and Fibonacci Sequence. An experiment was previously conducted to relate the Fibonacci Sequence with neural networks.

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As identical algorithms are utilized by the human brain and Neural Networks, the experiment was aiming to prove that the usage of Fibonacci Numbers as weights and the golden ratio as a learning rate would be able to enhance learning curve performance. By chance there was a performance improvement, this supports the fact that Neural Networks' algorithms represent its counterpart in nature. In LabVIEW, two same Neural Networks were coded with a slight change of one possessing unmethodically chosen weights, and the other (the modified one) possessing weights produced from Fibonacci numbers. The results expressed that the Neural Network coded with Fibonacci weights had a learning curve with a higher gradient. Under these conditions, this increased performance involving the neural algorithm forms the suggestion that this formula truly represents its counterpart in nature or vice versa. The application of the golden ratio in artificial intelligence (AI) is an interesting suggestion. The foundation problem of AI such as in designing algorithms for recognizing naturally occurring phenomena, such as human body movements and face recognition, AI effectively helps to create the algorithm and solution for the problem. If the formula is the natural counterpart's simulation, then in nature lies the weights of Fibonacci numbers. Following is an example of the application of the Fibonacci Sequence in AI.

Face Detection

Within the computer, vision field lies a foundational research topic known as face detection. Majority of face-related applications including face tracking and face recognition work under the assumption that the region of the face is flawlessly recognized. However, because of the lack of universal criteria in the literature, it is challenging to accurately evaluate face detection algorithms' performance. A paper by Hassaballah et al. (2013) proposed that the Golden Ratio could be utilized to evaluate face detection algorithms [36]. The Golden Ratio is useful in the sense that it estimates the size of the face in accordance with the distance between the centers of the eyes. A precise estimation is made of the size of the human face based on this distance. The experiment looked at the relationship between the inter-ocular distance or mouth width, as well as the width or height of the face, which are important values to consider when trying to estimate the human face size. Correct detection can be defined through the usage of the relative error measure in accordance with the distance between the estimated and expected eye positions. The detection is under the consideration of accuracy if the relative error is equal to or less than 0.25. In other words, the highest deviation from the actual eye center positions allowed is ½ of an eye width. The Golden Ratio is also utilized to change the located region of face detection from a square to a rectangle. This change is done by utilizing the Golden Ratio principle, in which the width and height of the square region detection must fulfill $h/w=1$. In this case, the height and width of the converted region must be able to fulfill $h/w=1.618$, or at the very least, a value close to this ratio to obtain a flawless face definition. At the end of the experiment, it was concluded that the golden ratio made it possible to define what is a face by utilizing certain parts of the human face, namely the eyes. It was proved that this proposed method was indeed more realistic and accurate to evaluate face detection. In another research, researchers proposed a new scheme to align the face by using a support vector machine and Viola-jones to identify the face with the extraction technique of the golden ration function. The technique was more realistic, effective and accurate compared with other face detection techniques [37]. Similarly, the golden ratio was applied in another research to classify the shapes among three different groups of people. The authors worked with three different nationalities parameters of the width of the face, physiognomical facial height and physiognomical facial index was considered and the facial morphology was identified with a golden ratio [38].

V. GOLDEN RATIO IN QUANTUM COMPUTATIONS

Understanding of the physical world can be greatly improved through symmetry, which fundamentally has a significant job in this understanding. The most curiosity-inducing type os E8, which essentially is an extraordinarily pleasing 8D diamond-like lattice of spheres, in which surrounding each sphere is another 240 spheres. Alexander Zamolodchikov, a Russian physicist 1988, proved that E8 symmetry, under specific conditions, was capable of describing the spectrum of spin excitations often witnessed in using ferromagnets. These chains of spins are only able to come into contact with two of their closest neighbors. Below a certain temperature, neighboring spins have the tendency of aligning perpendicular to the chain's direction in one or two directions.

Perpendicularly to the spins, if a magnetic field were to be applied, the spins are encouraged to spontaneously tunnel or flip, between down and up. These types of fluctuations have the capability of propagating through a material, similar to a particle, and are hence called quasiparticles. At Rutherford Appleton Laboratory, the University of Bristol, and Helmholtz Zentrum Berlin, Radu Coldea of Oxford University, as well as his colleagues, had measured several quasiparticles' energies. By cooling a sample of cobalt niobate to 40 mK, followed by releasing neurons at it to form quasiparticles, Coldea and colleagues were able to conduct their experiment. As this occurs, the energy and spin of the dispersed neurons relatively change to the incident beam by a quantity that is utilized to calculate the quasiparticles' energies. This experiment was conducted at zero magnetic fields, in which five quasiparticles were spotted.

Repeating this experiment in a magnetic field, the field's strength raised to a quantum critical value of 5.5 T, in which the energies' ratio of the first and second quasiparticles came close to 1.618. Described as the "Golden Ratio," this number is what should be measured by the situation that the quasiparticles are described as E8 – this is a prediction made by Zamolodchikov. The quantum critical field where E8 should emerge could not be well studied due to the fact that the lowest energy quasiparticles that could be resolved were above 5 T. Eight quasiparticles are predicted by E8, while only five could be found due to the Golden Ratio not being achieved. This could not be achieved due to the energies of neutrons overlapping a region controlled by continuum scattering, under the involvement of two or more quasiparticles.

Time in Quantum Physics and Phi

In Quantum physics, time could be correlated with the Golden Ratio, which appears to be reappearing in the Quantum Physics Model. An electron possesses an intrinsic rotational spin or motion, as well as an electric charge, and as a result, behaves similarly to a tiny bar magnet, also said to possess a magnetic moment. Due to electrons possessing mass, it behaves similarly to a spinning top, also said to possess spin angular momentum.

The electron's g factor can be defined as the ratio of its magnetic moment to its spin angular momentum. This g-factor is caused by the expanding of space-time as the electron rotates at light's speed.

The electron g-factor is around: gfactore = -2 / sin (∅)

And the proton g-factor is around: gfactorp = 2∅ / sin (1/∅)

As a result, there is the appearance that Phi, or the Golden Ratio, is a constant that time manufactured. According to the National Institute of Standards and Technology, these gfactor constants are stated as can be seen below:

Factor	gfactor electron	gfactor proton
Phi-based	-2.0022334732293	5.5848781529840
Per NIST	-2.0023193043718	5.5856947010000
Variance	-0.004287%	-0.014619%

Although the approach based on the Golden Ratio is not identical to the NIST constant, through the usage of quantum level constants, a variance between empirical and theoretical work will always be present. This may be caused by stray background radiations, the irregularities in the test equipment's metal, and more. As a result, a certain degree of statistical spread is always shown in empirical measurement results.

This overview displays the fact and the post elucidations on gold proportions have been implemented in wide range of disciplines and natural phenomenon. These are either man-made or occurred naturally and some of them are not realized by everyone. These things are made creatures easily and should follow in the new creature. Further, if any design follows this basis, there will be a great outcome and several creatures will be formed.

REFERENCES

[1] D. C. Rajput, "Golden Ratio," J. Adv. Math., vol. 20, pp. 19–42, 2021, doi: 10.24297/jam.v20i.8945.

[2] A. F. Nematollahi, A. Rahiminejad, and B. Vahidi, "A novel meta-heuristic optimization method based on golden ratio in nature," Soft Comput., vol. 24, no. 2, pp. 1117–1151, 2020, doi: 10.1007/s00500-019-03949-w.

[3] N. Giansiracusa, "Fibonacci, golden ratio, and vector bundles," Mathematics, vol. 9, no. 4, p. 426, 2021, doi: 10.3390/math9040426.

[4] E. Nexo, "The Pentagon Tells it all. The Golden Ratio and Fibonacci Numbers," J. Adv. Math. Comput. Sci., pp. 35–49, 2021, doi: 10.9734/jamcs/2021/v36i1130416.

[5] K. Yalta, S. Ozturk, and E. Yetkin, "Golden Ratio and the heart: A review of divine aesthetics," International Journal of Cardiology, vol. 214. pp. 107–112, 2016, doi: 10.1016/j.ijcard.2016.03.166.

[6] T. O. Omotehinwa, "Fibonacci Numbers and Golden Ratio in Mathematics and Science," Int. J. Comput. Inf. Technol., vol. 2, no. 4, pp. 630–638, 2013.

[7] B. Kaygn, B. Balçin, C. Yildiz, and S. Arslan, "The effect of teaching the subject of Fibonacci numbers and golden ratio through the history of mathematics," in Procedia - Social and Behavioral Sciences, 2011, vol. 15, pp. 961–965, doi: 10.1016/j.sbspro.2011.03.221.

[8] K. Shekhawat, "Why golden rectangle is used so often by architects: A mathematical approach," Alexandria Eng. J., vol. 54, no. 2, pp. 213–222, 2015, doi: 10.1016/j.aej.2015.03.012.

[9] J. S. Duan, "Shrinkage Points of Golden Rectangle, Fibonacci Spirals, and Golden Spirals," Discret. Dyn. Nat. Soc., vol. 2019, p. 3149602, 2019, doi: 10.1155/2019/3149602.

[10] J. Fernández-Llebrez and J. M. Fran, "The Church in The Hague by Aldo van Eyck: The Presence of the Fibonacci Numbers and the Golden Rectangle in the Compositional Scheme of the Plan," Nexus Netw. J., vol. 15, no. 2, pp. 303–323, 2013, doi: 10.1007/s00004-013-0151-y.

[11] S. Sivankutty, V. Tsvirkun, O. Vanvincq, G. Bouwmans, E. R. Andresen, and H. Rigneault, "Nonlinear imaging through a Fermat's golden spiral multicore fiber," Opt. Lett., vol. 43, no. 15, pp. 3638–3641, 2018, doi: 10.1364/ol.43.003638.

[12] N. K. Paul and H. K. Saikia, "A generalization of Fibonacci sequence," Proyecciones, vol. 39, no. 6, p. 1394=1405, 2020, doi: 10.22199/issn.0717-6279-2020-06-0085.

[13] N. K. Paul and H. K. Saikia, "Some generalized results related to Fibonacci sequence," Proyecciones, vol. 40, no. 3, pp. 606–617, 2021, doi: 10.22199/ISSN.0717-6279-4269.

[14] S. Sinha, "The Fibonacci Numbers and Its Amazing Applications," Int. J. Eng. Sci. Invent., vol. 6, no. 9, pp. 7–14, 2017.

[15] D. Persaud and J. P. O'Leary, "Fibonacci Series, Golden Proportions, and the Human Biology," Austin J Surg., vol. 2, no. 5, p. 1066, 2015.

[16] E. Petekkaya et al., "Evaluation of the Golden Ratio in Nasal Conchae for Surgical Anatomy," Ear, Nose Throat J., vol. 100, no. 1, pp. NP57–NP61, 2021, doi: 10.1177/0145561319862786.

[17] G. Yetkin, N. Sivri, K. Yalta, and E. Yetkin, "Golden Ratio is beating in our heart," Int. J. Cardiol., vol. 168, no. 5, pp. 4926–4927, 2013, doi: 10.1016/j.ijcard.2013.07.090.

[18] S. H. Larsen, "Dna structure and the golden ratio revisited," Symmetry (Basel)., vol. 13, no. 10, p. 1949, 2021, doi: 10.3390/sym13101949.

[19] C. Gueunet, P. Fortin, J. Jomier, and J. Tierny, "Task-Based Augmented Contour Trees with Fibonacci Heaps," IEEE Trans. Parallel Distrib. Syst., vol. 30, no. 8, pp. 1889–1905, 2019, doi: 10.1109/TPDS.2019.2898436.

[20] T. Batista Da Silveira, E. Mendes Duque, S. J. Ferzoli Guimaraes, H. Torres Marques-Neto, and H. Cota De Freitas, "Proposal of Fibonacci Heap in the Dijkstra Algorithm for Low-power Ad-hoc Mobile Transmissions," IEEE Lat. Am. Trans., vol. 18, no. 3, pp. 623–630, 2020, doi: 10.1109/TLA.2020.9082735.

[21] M. L. Fredman and R. E. Tarjan, "Fibonacci heaps and their uses in improved network optimization algorithms," J. ACM, vol. 34, no. 3, pp. 596–615, 1987, doi: 10.1145/28869.28874.

[22] M. L. Fredman and R. E. Tarjan, "Fibonacci heaps and theie uses in improved network optimization algorithms," in Proceedings - Annual IEEE Symposium on Foundations of Computer Science, FOCS, 1984, vol. 1984-Octob, pp. 338–346, doi: 10.1109/sfcs.1984.715934.

[23] J. H. Zhang, X. Y. Wei, L. Hu, and J. G. Ma, "A MPPT method based on improved fibonacci search photovoltaic array," Teh. Vjesn., vol. 26, no. 1, pp. 163–170, Feb. 2019, doi: 10.17559/TV-20180721153103.

[24] C. Y. Chong, S. K. Leow, and H. S. Sim, "Generalized fibonacci search method in one-dimensional unconstrained non-linear optimization," Pertanika J. Sci. Technol., vol. 29, no. 2, pp. 1017–1039, 2021, doi: 10.47836/pjst.29.2.17.

[25] W. Giernacki, D. Horla, T. Báča, and M. Saska, "Real-time model-free minimum-seeking autotuning method for unmanned aerial vehicle controllers based on fibonacci-search algorithm," Sensors (Switzerland), vol. 19, no. 2, p. 312, Jan. 2019, doi: 10.3390/s19020312.

[26] T. Dong, H. Zhang, and F. Zeng, "A Beamformer Design Based on Fibonacci Branch Search," Prog. Electromagn. Res. B, vol. 88, pp. 73–95, 2020, doi: 10.2528/PIERB20033103.

[27] A. J. Raphael and V. Sundaram, "Secured Communication through Fibonacci Numbers and Unicode Symbols," vol. 3, no. 4, pp. 1–5, 2012.

[28] S. Lagheliel, A. Chillali, and A. A. Mokhtar, "New encryption scheme using k-Fibonacci-like sequence," Asian-European J. Math., vol. 15, no. 2, p. 2250037, 2022, doi: 10.1142/S1793557122500371.

[29] B. Aboushosha, R. A. Ramadan, A. D. Dwivedi, A. El-Sayed, and M. M. Dessouky, "SLIM: A lightweight block cipher for internet of health things," IEEE Access, vol. 8, pp. 203747–203757, 2020, doi: 10.1109/ACCESS. 2020.3036589.

[30] I. Shahzad, Q. Mushtaq, and A. Razaq, "Construction of New S-Box Using Action of Quotient of the Modular Group for Multimedia Security," Secur. Commun. Networks, vol. 2019, p. 2847801, 2019, doi: 10.1155/2019/2847801.

[31] K. Mohamed, F. Hani Hj Mohd Ali, S. Ariffin, N. Hafiza Zakaria, and M. Nazran Mohammed Pauzi, "An Improved AES S-box Based on Fibonacci Numbers and Prime Factor," Int. J. Netw. Secur., vol. 20, no. 6, p. 1206, 2018, doi: 10.6633/IJNS.201811.

[32] M. V. Ahamad, M. U. Siddiqui, M. Masroor, and U. Fatima, "An improved Playfair encryption technique using fibonacci series generated secret key," Int. J. Eng. Technol., vol. 7, no. 4, pp. 347–351, 2018, doi: 10.14419/ijet.v 7i4.5.20104.

[33] B. S. Tarle and G. L. Prajapati, "On the information security using Fibonacci series," Int. Conf. Work. Emerg. Trends Technol. 2011, ICWET 2011 - Conf. Proc., no. January 2011, pp. 791–797, 2011, doi: 10.1145/1980022.1980195.

[34] R. M E and R. K C, "Application of Classical Encryption Techniques for Securing Data- A Threaded Approach," Int. J. Cybern. Informatics, vol. 4, no. 2, pp. 125–132, 2015, doi: 10.5121/ijci.2015.4212.

[35] S. Saxena and A. Sharma, "Encryption and decryption using hybrid cryptography techniques and multi-level steganography," Int. J. Innov. Technol. Explor. Eng., vol. 8, no. 9, pp. 40–45, 2019, doi: 10.35940/ijitee.i7472.078919.

[36] M. Hassaballah, K. Murakami, and S. Ido, "Face detection evaluation: A new approach based on the golden ratio Φ," Signal, Image Video Process., vol. 7, no. 2, pp. 307–316, 2013, doi: 10.1007/s11760-011-0239-3.

[37] P. S. Gaikwad and V. B. Kulkarni, "Face Recognition Using Golden Ratio for Door Access Control System," in Lecture Notes in Electrical Engineering, 2021, vol. 703, pp. 209–231, doi: 10.1007/978-981-15-8391-9_16.

[38] V. Packiriswamy, P. Kumar, and M. Rao, "Identification of facial shape by applying golden ratio to the facial measurements: An interracial study in Malaysian population," N. Am. J. Med. Sci., vol. 4, no. 12, pp. 624–629, 2012, doi: 10.4103/1947-2714.104312.